ReSolveD: Shorter Signatures from Regular Syndrome Decoding and VOLE-in-the-Head

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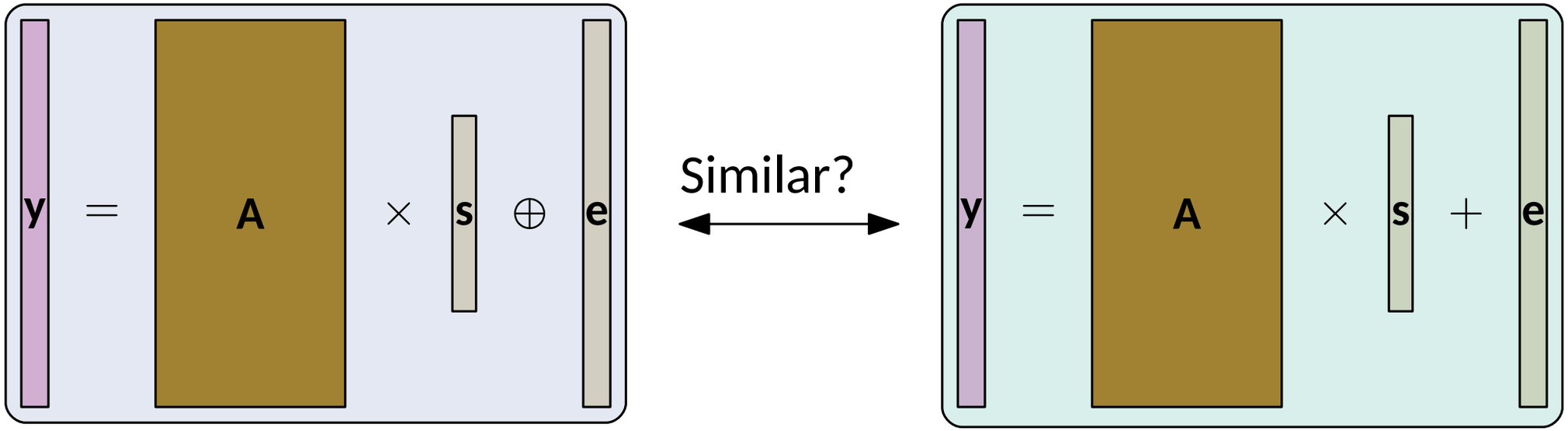
In Submission

Motivations

- Consider **L**earning **P**arity with **N**oise (aka, **S**yndrome **D**ecoding.)
- $(\mathbf{A}, \mathbf{y}) \approx (\mathbf{A}, U)$, for low-Hamming weight $\mathbf{s}, \mathbf{e}$



LPN: over $\mathbb{F}_2$ LWE: over $\mathbb{F}_q$

- We have very efficient LWE-based signatures ([GPV08, Lyu12, ...])
- How about LPN-based?

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- So what now?

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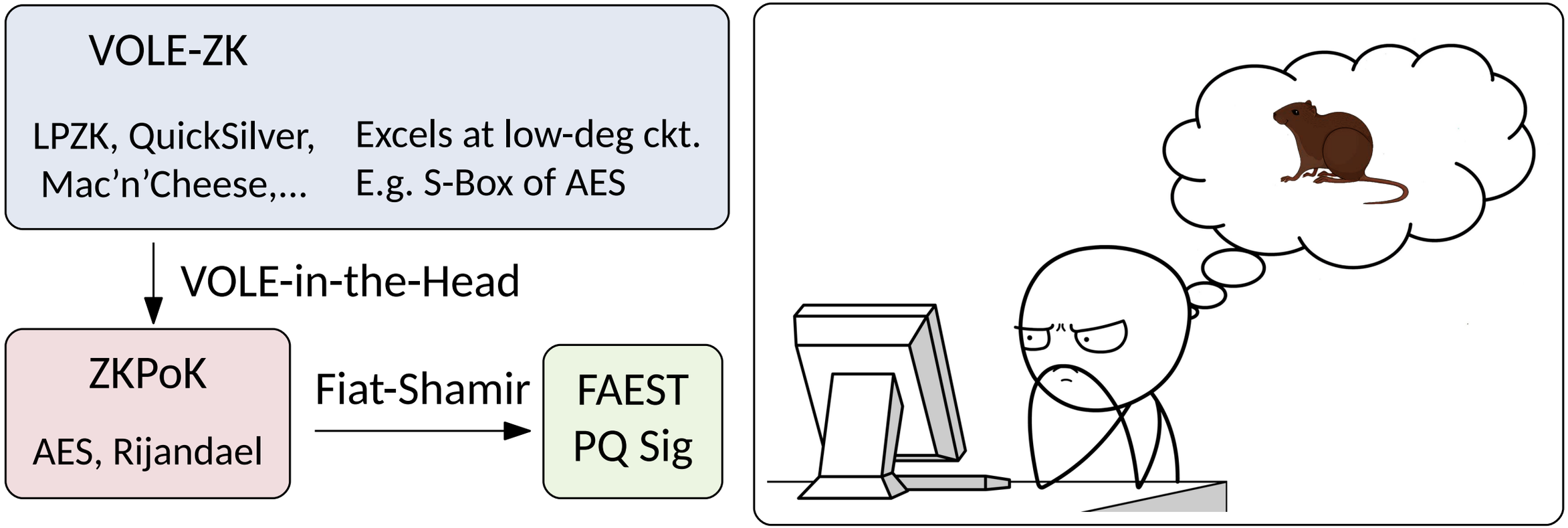
- Luckily, arithmetic over $\mathrm{GF}(2)$ is friendly to MPC

- A number of existing works with increasingly better efficiency…
- e.g. [GPS21, FJR21, BGKM22, FJR22, CCR23]

Exciting New Tools: VOLEitH

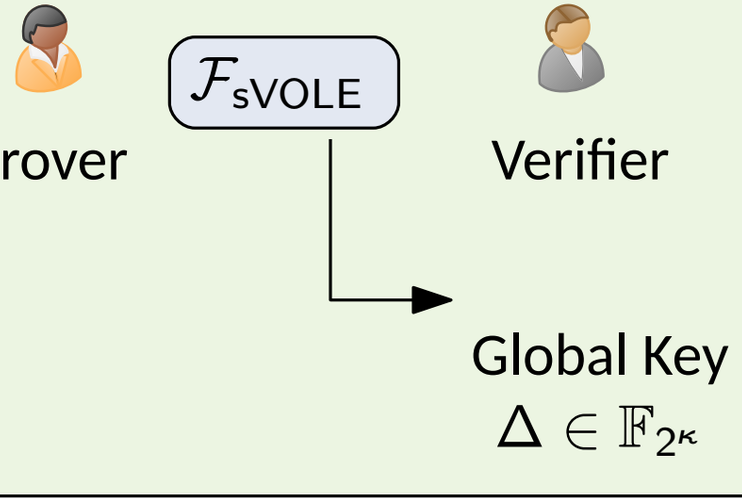
- Provides new method to convert VOLE-ZK to ZKPoK



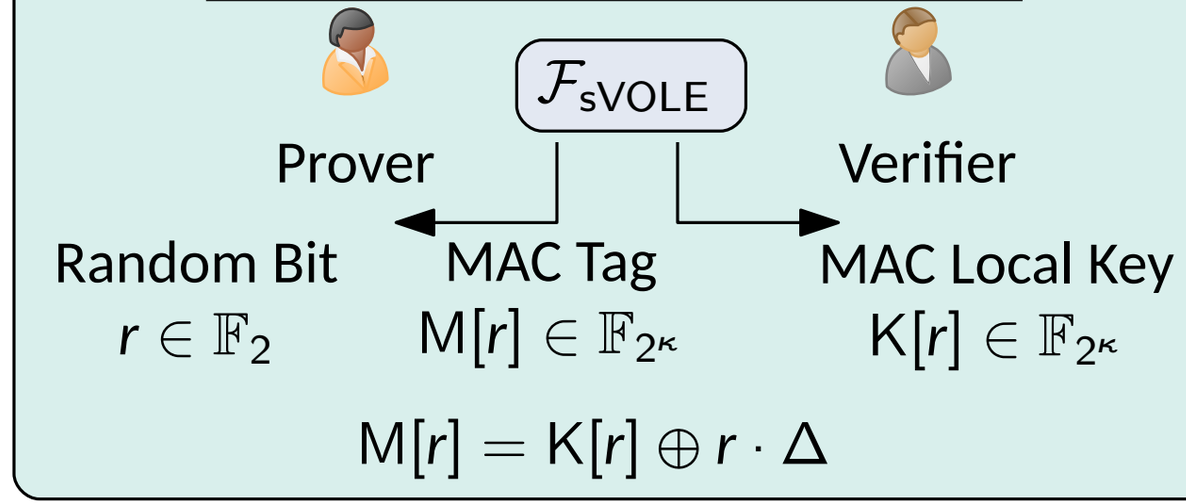
- BBD+23: Publicly Verifiable Zero-Knowledge and Post-Quantum Signatures from VOLE-in-the-Head (Crypto 2023)

(The Only) Preliminary

Initialization

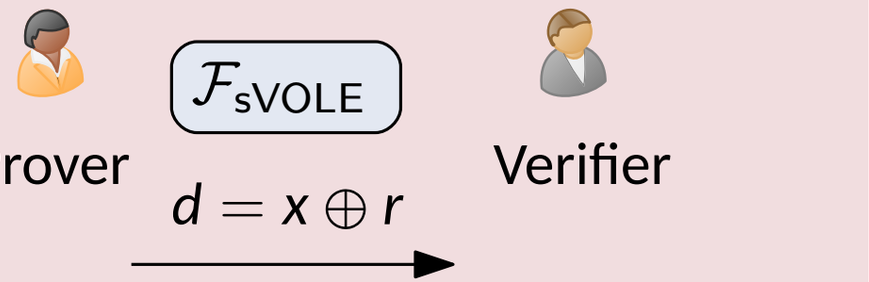


Generating Random Correlations



$$M[r] = K[r] \oplus r \cdot \Delta$$

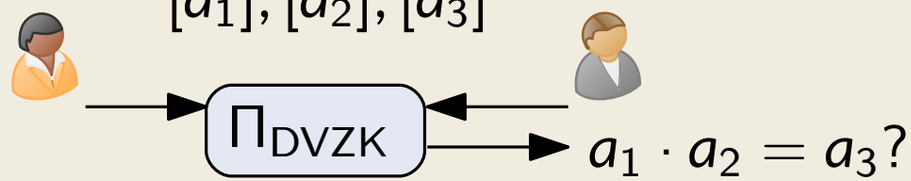
Input Commitment



commit to x by sending diff.

$$M[x] = M[r] \qquad K[x] = K[r] \oplus d \cdot \Delta$$

Proving AND Relation

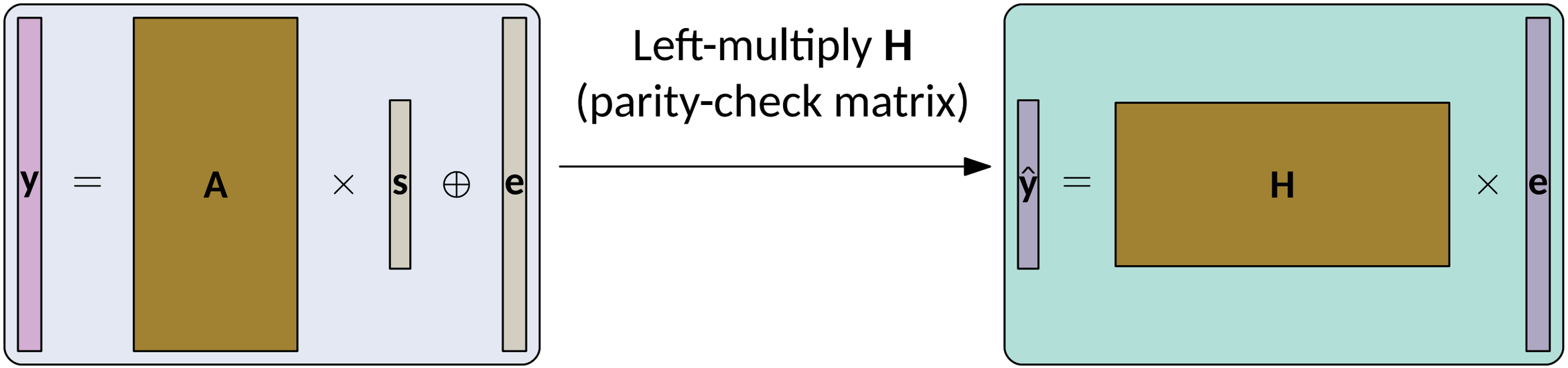


$$K_1K_2 \oplus \Delta K_3 = (a_1a_2 \oplus a_3)\Delta^2$$
$$\oplus\,(a_1M_2 \oplus a_2M_1 \oplus M_3)\Delta \oplus M_1M_2$$

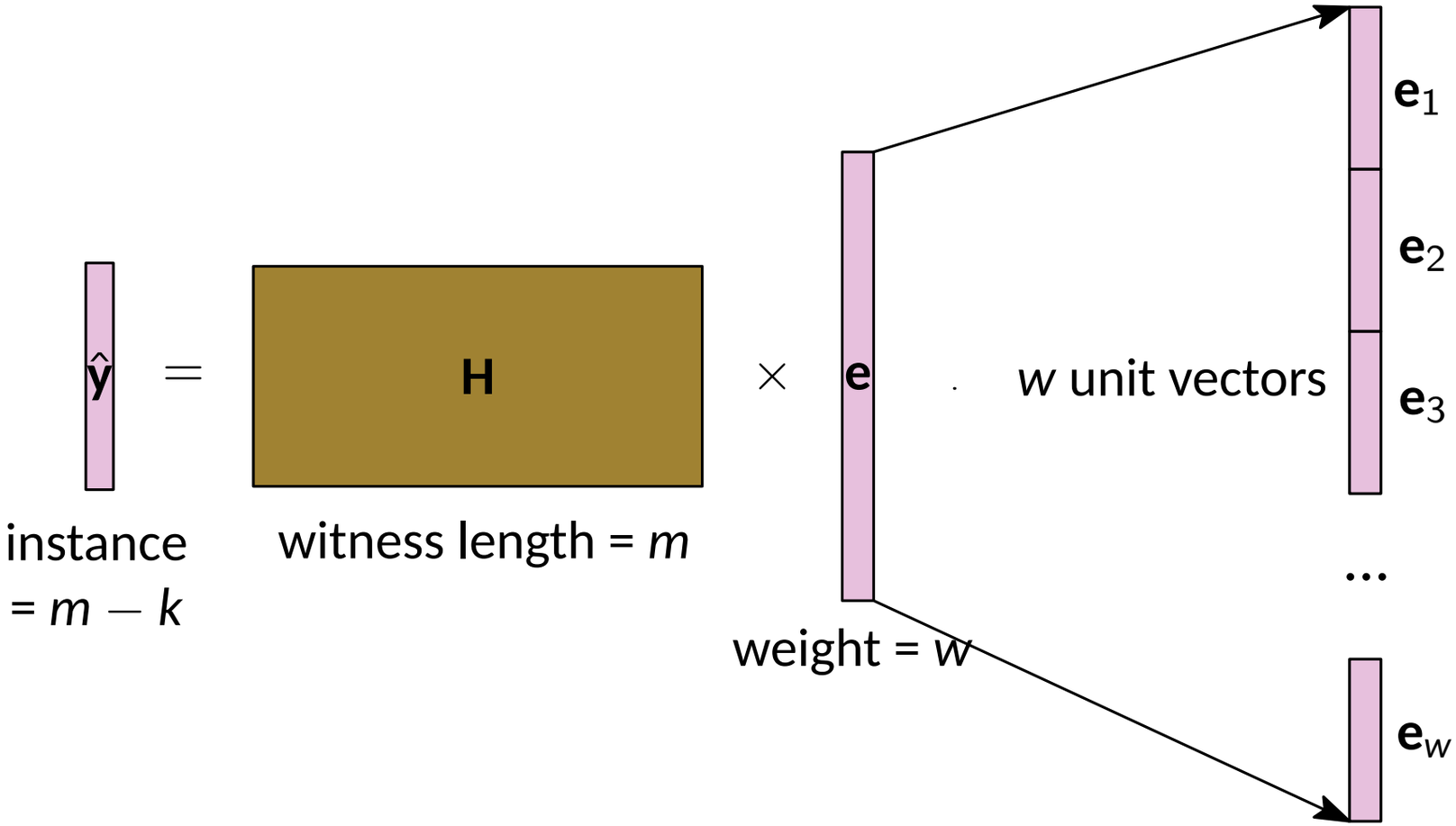
Let’s try it on LPN/SD

- Main technical challenge:
- Proving low-hamming weight of **e**

- Step 1: Convert LPN to dual form

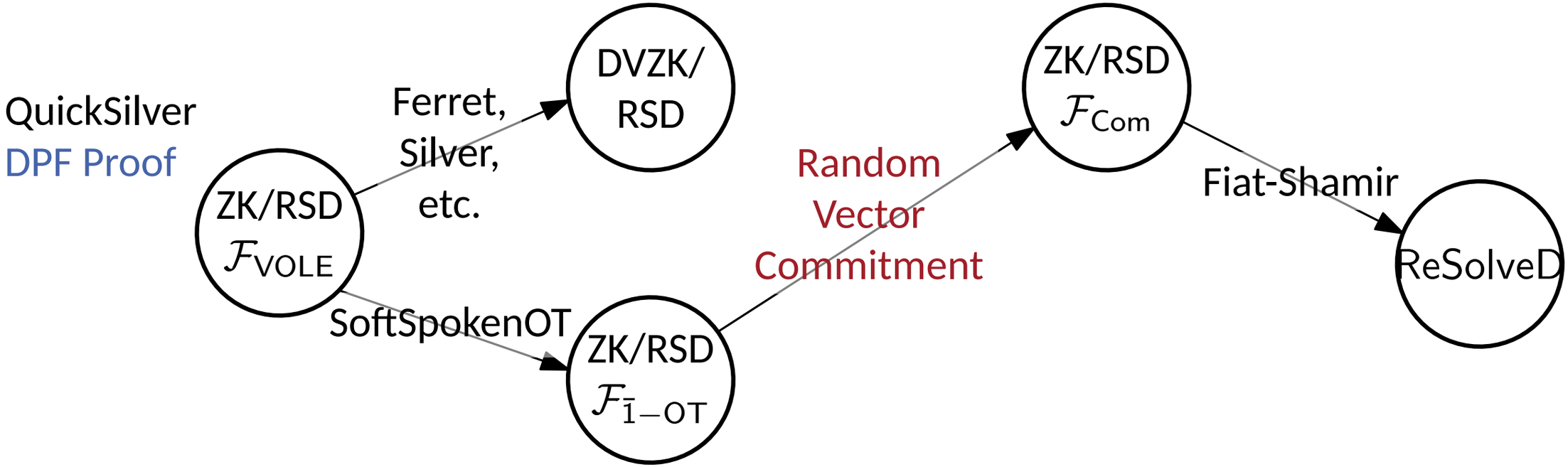


To Make our Life Easier...



- Systematic Form: $\mathbf{H} = [\mathbf{I}_{m-k} \| \mathbf{H_B}]$
- $\hat{\mathbf{y}} = \mathbf{H} \cdot \mathbf{e} = \mathbf{e}_A + \mathbf{H_B} \cdot \mathbf{e}_B$
- We only commit and get $[\mathbf{e}_B]$ and reconstruct $[\mathbf{e}] = [\hat{\mathbf{y}} - \mathbf{H_B} \cdot \mathbf{e}_B \| \mathbf{e}_B]$

Contributions



- Contribution 1: Combine DPF proof with VOLE-in-the-Head
- Contribution 2: Use half-tree to optimize computational performance

Thanks for your listening

Long Live the Code-based Crypto